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# Division of complex numbers

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In this unit we are going to look at how to divide a complex number by another complex number.

Division of complex numbers relies on two important principles. The first is that multiplying a complex number by its conjugate produces a purely real number. The second principle is that both the numerator and denominator of a fraction can be multiplied by the same number, and the value of the fraction will remain unchanged.

For example, starting with the fraction $\frac{1}{2}$, we can multiply both top and bottom by 5 to give $\frac{5}{10}$, and the value of this is the same as $\frac{1}{2}$. We say that $\frac{1}{2}$ and $\frac{5}{10}$ are equivalent fractions.

**Example.** Suppose we want to divide the complex number $(4+7\mathrm{i})$ by $(1-3\mathrm{i})$, that is we want to find

$$\frac{4+7\mathrm{i}}{1-3\mathrm{i}}$$

We won't change the value of this fraction if we multiply both numerator and denominator by the same value. We multiply by the conjugate of the denominator, which is $1+3\mathrm{i}$, and then simplify.

$$\begin{aligned}\frac{(4+7\mathrm{i})(1+3\mathrm{i})}{(1-3\mathrm{i})(1+3\mathrm{i})} &= \frac{4+12\mathrm{i}+7\mathrm{i}+21\mathrm{i}^2}{1+3\mathrm{i}-3\mathrm{i}-9\mathrm{i}^2}\\ &= \frac{-17+19\mathrm{i}}{10}\\ &= -\frac{17}{10}+\frac{19}{10}\mathrm{i}\\ &= -1.7+1.9\mathrm{i}\end{aligned}$$

**Example.** Suppose we want to divide the complex number $(2-5\mathrm{i})$ by $(-4+3\mathrm{i})$, that is we want to find

$$\frac{2-5\mathrm{i}}{-4+3\mathrm{i}}$$

We multiply by the conjugate of the denominator, which is $-4-3\mathrm{i}$, and then simplify.

$$\begin{aligned}\frac{(2-5\mathrm{i})(-4-3\mathrm{i})}{(-4+3\mathrm{i})(-4-3\mathrm{i})} &= \frac{-8-6\mathrm{i}+20\mathrm{i}+15\mathrm{i}^2}{16+12\mathrm{i}-12\mathrm{i}-9\mathrm{i}^2}\\ &= \frac{-23+14\mathrm{i}}{25}\\ &= -\frac{23}{25}+\frac{14}{25}\mathrm{i}\\ &= -0.92+0.56\mathrm{i} \qquad (\text{2dp})\end{aligned}$$

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In the next unit we will introduce the Argand Diagram, which is a graphical way of representing complex numbers.